Solution to Chapter 4 Problems

Problem 4.1

1) Since $\mathcal{F}[\text{sinc}(400t)] = \frac{1}{400}\Pi(\frac{f}{400})$, the bandwidth of the message signal is $W = 200$ and the resulting modulation index

$$\beta_f = \frac{k_f \max[|m(t)|]}{W} = \frac{k_f 10}{W} = 6 \Longrightarrow k_f = 120$$

Hence, the modulated signal is

$$\begin{aligned} u(t) &= A\cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau) \\ &= 100\cos(2\pi f_c t + +2\pi 1200 \int_{-\infty}^{t} \text{sinc}(400\tau)d\tau) \end{aligned}$$

2) The maximum frequency deviation of the modulated signal is

$$\Delta f_{\max} = \beta_f W = 6 \times 200 = 1200$$

3) Since the modulated signal is essentially a sinusoidal signal with amplitude $A = 100$, we have

$$P = \frac{A^2}{2} = 5000$$

4) Using Carson's rule, the effective bandwidth of the modulated signal can be approximated by

$$B_c = 2(\beta_f + 1)W = 2(6+1)200 = 2800 \text{ Hz}$$

Problem 4.2

1) The maximum phase deviation of the PM signal is

$$\Delta\phi_{\max} = k_p \max[|m(t)|] = k_p$$

The phase of the FM modulated signal is

$$\begin{aligned} \phi(t) &= 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau = 2\pi k_f \int_{0}^{t} m(\tau)d\tau \\ &= \begin{cases} 2\pi k_f \int_0^t \tau d\tau = \pi k_f t^2 & 0 \le t < 1 \\ \pi k_f + 2\pi k_f \int_1^t d\tau = \pi k_f + 2\pi k_f (t-1) & 1 \le t < 2 \\ \pi k_f + 2\pi k_f - 2\pi k_f \int_2^t d\tau = 3\pi k_f - 2\pi k_f (t-2) & 2 \le t < 3 \\ \pi k_f & 3 \le t \end{cases} \end{aligned}$$

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The maximum value of $\phi(t)$ is achieved for $t = 2$ and is equal to $3\pi k_f$. Thus, the desired relation between k_p and k_f is

$$k_p = 3\pi k_f$$

2) The instantaneous frequency for the PM modulated signal is

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d}{dt}\phi(t) = f_c + \frac{1}{2\pi}k_p\frac{d}{dt}m(t)$$

For the $m(t)$ given in Fig. P-4.2, the maximum value of $\frac{d}{dt}m(t)$ is achieved for t in [0, 1] and it is equal to one. Hence,

$$\max(f_i(t)) = f_c + \frac{1}{2\pi}$$

For the FM signal $f_i(t) = f_c + k_f m(t)$. Thus, the maximum instantaneous frequency is

$$\max(f_i(t)) = f_c + k_f = f_c + 1$$

Problem 4.3

For an angle modulated signal we have $x(t) = A_c\cos(2\pi f_c t + \phi(t))$, therefore The lowpass equivalent of the signal is $x_l(t) = A_c e^{j\phi(t)}$ with Envelope A_c and phase $\pi(t)$ and in phase an quadrature components $A_c\cos(\phi(t))$ and $A_c\sin(\phi(t))$, respectively. Hence we have the following

$$\text{PM}\quad\begin{cases} A_c & \text{envelope} \\ k_p m(t) & \text{phase} \\ A_c\cos\left(k_p m(t)\right) & \text{in-phase comp.} \\ A_c\sin\left(k_p m(t)\right) & \text{quadrature comp.} \end{cases}\qquad \text{FM}\quad\begin{cases} A_c & \text{envelope} \\ 2\pi k_f\int_{-\infty}^{t} m(\tau)\,d\tau & \text{phase} \\ A_c\cos\left(2\pi k_f\int_{-\infty}^{t} m(\tau)\,d\tau\right) & \text{in-phase comp.} \\ A_c\sin\left(2\pi k_f\int_{-\infty}^{t} m(\tau)\,d\tau\right) & \text{quadrature comp.} \end{cases}$$

Problem 4.4

1) Since an angle modulated signal is essentially a sinusoidal signal with constant amplitude, we have

$$P = \frac{A_c^2}{2} \Longrightarrow P = \frac{100^2}{2} = 5000$$

The same result is obtained if we use the expansion

$$u(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\cos(2\pi(f_c + nf_m)t)$$

along with the identity

$$J_0^2(\beta) + 2\sum_{n=1}^{\infty} J_n^2(\beta) = 1$$

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2) The maximum phase deviation is

$$\Delta\phi_{\max} = \max|4\sin(2000\pi t)| = 4$$

3) The instantaneous frequency is

$$\begin{aligned} f_i &= f_c + \frac{1}{2\pi}\frac{d}{dt}\phi(t) \\ &= f_c + \frac{4}{2\pi}\cos(2000\pi t)2000\pi = f_c + 4000\cos(2000\pi t) \end{aligned}$$

Hence, the maximum frequency deviation is

$$\Delta f_{\max} = \max|f_i - f_c| = 4000$$

4) The angle modulated signal can be interpreted both as a PM and an FM signal. It is a PM signal with phase deviation constant $k_p = 4$ and message signal $m(t) = \sin(2000\pi t)$ and it is an FM signal with frequency deviation constant $k_f = 4000$ and message signal $m(t) = \cos(2000\pi t)$.

Problem 4.5

The modulated signal can be written as

$$u(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\cos(2\pi(f_c + nf_m)t)$$

The power in the frequency component $f = f_c + kf_m$ is $P_k = \frac{A_c^2}{2}J_n^2(\beta)$. Hence, the power in the carrier is $P_{\text{carrier}} = \frac{A_c^2}{2}J_0^2(\beta)$ and in order to be zero the modulation index β should be one of the roots of $J_0(x)$. The smallest root of $J_0(x)$ is found from tables to be equal 2.404. Thus,

$$\beta_{\min} = 2.404$$

Problem 4.6

1) If the output of the narrowband FM modulator is,

$$u(t) = A\cos(2\pi f_0 t + \phi(t))$$

then the output of the upper frequency multiplier ($\times n_1$) is

$$u_1(t) = A\cos(2\pi n_1 f_0 t + n_1\phi(t))$$

After mixing with the output of the second frequency multiplier $u_2(t) = A\cos(2\pi n_2 f_0 t)$ we obtain the signal

$$\begin{aligned} y(t) &= A^2\cos(2\pi n_1 f_0 t + n_1\phi(t))\cos(2\pi n_2 f_0 t) \\ &= \frac{A^2}{2}(\cos(2\pi(n_1+n_2)f_0 + n_1\phi(t)) + \cos(2\pi(n_1-n_2)f_0 + n_1\phi(t))) \end{aligned}$$

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The bandwidth of the signal is $W = 15$ KHz, so the maximum frequency deviation is $\Delta f = \beta_f W = 0.1 \times 15 = 1.5$ KHz. In order to achieve a frequency deviation of $f = 75$ KHz at the output of the wideband modulator, the frequency multiplier n_1 should be equal to

$$n_1 = \frac{f}{\Delta f} = \frac{75}{1.5} = 50$$

Using an up-converter the frequency modulated signal is given by

$$y(t) = \frac{A^2}{2}\cos(2\pi(n_1+n_2)f_0 + n_1\phi(t))$$

Since the carrier frequency $f_c = (n_1+n_2)f_0$ is 104 MHz, n_2 should be such that

$$(n_1+n_2)100 = 104 \times 10^3 \Longrightarrow n_1+n_2 = 1040 \text{ or } n_2 = 990$$

2) The maximum allowable drift (d_f) of the 100 kHz oscillator should be such that

$$(n_1+n_2)d_f = 2 \Longrightarrow d_f = \frac{2}{1040} = .0019 \text{ Hz}$$

Problem 4.7

The modulated PM signal is given by

$$\begin{aligned} u(t) &= A_c\cos(2\pi f_c t + k_p m(t)) = A_c \mathrm{Re}\left[e^{j2\pi f_c t}e^{jk_p m(t)}\right] \\ &= A_c \mathrm{Re}\left[e^{j2\pi f_c t}e^{jm(t)}\right] \end{aligned}$$

The signal $e^{jm(t)}$ is periodic with period $T_m = \frac{1}{f_m}$ and Fourier series expansion

$$\begin{aligned} c_n &= \frac{1}{T_m}\int_0^{T_m} e^{jm(t)}e^{-j2\pi n f_m t}dt \\ &= \frac{1}{T_m}\int_0^{\frac{T_m}{2}} e^{j}e^{-j2\pi n f_m t}dt + \frac{1}{T_m}\int_{\frac{T_m}{2}}^{T_m} e^{-j}e^{-j2\pi n f_m t}dt \\ &= -\frac{e^{j}}{T_m j2\pi n f_m}e^{-j2\pi n f_m t}\Big|_0^{\frac{T_m}{2}} - \frac{e^{-j}}{T_m j2\pi n f_m}e^{-j2\pi n f_m t}\Big|_{\frac{T_m}{2}}^{T_m} \\ &= \frac{(-1)^n - 1}{2\pi n}j(e^{j} - e^{-j}) = \begin{cases} 0 & n = 2l \\ \frac{2}{\pi(2l+1)}\sin(1) & n = 2l+1 \end{cases} \end{aligned}$$

Hence,

$$e^{jm(t)} = \sum_{l=-\infty}^{\infty}\frac{2}{\pi(2l+1)}\sin(1)e^{j2\pi l f_m t}$$

and

$$\begin{aligned} u(t) &= A_c\mathrm{Re}\left[e^{j2\pi f_c t}e^{jm(t)}\right] = A_c\mathrm{Re}\left[e^{j2\pi f_c t}\sum_{l=-\infty}^{\infty}\frac{2}{\pi(2l+1)}\sin(1)e^{j2\pi l f_m t}\right] \\ &= A_c\sum_{l=-\infty}^{\infty}\left|\frac{2\sin(1)}{\pi(2l+1)}\right|\cos(2\pi(f_c + l f_m)t + \phi_l) \end{aligned}$$

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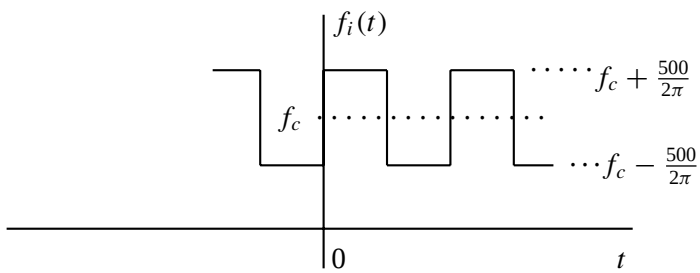
where $\phi_l = 0$ for $l \geq 0$ and $\phi_l = \pi$ for negative values of l.

Problem 4.8

1) The instantaneous frequency is given by

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d}{dt}\phi(t) = f_c + \frac{1}{2\pi}100m(t)$$

A plot of $f_i(t)$ is given in the next figure



2) The peak frequency deviation is given by

$$\Delta f_{\max} = k_f \max[|m(t)|] = \frac{100}{2\pi}5 = \frac{250}{\pi}$$

Problem 4.9

1) The modulation index is

$$\beta = \frac{k_f \max[|m(t)|]}{f_m} = \frac{\Delta f_{\max}}{f_m} = \frac{20 \times 10^3}{10^4} = 2$$

The modulated signal $u(t)$ has the form

$$\begin{aligned} u(t) &= \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\cos(2\pi(f_c + nf_m)t + \phi_n) \\ &= \sum_{n=-\infty}^{\infty} 100 J_n(2)\cos(2\pi(10^8 + n10^4)t + \phi_n) \end{aligned}$$

The power of the unmodulated carrier signal is $P = \frac{100^2}{2} = 5000$. The power in the frequency component $f = f_c + k10^4$ is

$$P_{f_c+kf_m} = \frac{100^2 J_k^2(2)}{2}$$

The next table shows the values of $J_k(2)$, the frequency $f_c + kf_m$, the amplitude $100J_k(2)$ and the power $P_{f_c+kf_m}$ for various values of k.

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Index k	$J_k(2)$	Frequency Hz	Amplitude $100J_k(2)$	Power $P_{f_c+kf_m}$
0	.2239	10^8	22.39	250.63
1	.5767	$10^8 + 10^4$	57.67	1663.1
2	.3528	$10^8 + 2 \times 10^4$	35.28	622.46
3	.1289	$10^8 + 3 \times 10^4$	12.89	83.13
4	.0340	$10^8 + 4 \times 10^4$	3.40	5.7785

As it is observed from the table the signal components that have a power level greater than 500 (= 10% of the power of the unmodulated signal) are those with frequencies $10^8 + 10^4$ and $10^8 + 2 \times 10^4$. Since $J_n^2(\beta) = J_{-n}^2(\beta)$ it is conceivable that the signal components with frequency $10^8 - 10^4$ and $10^8 - 2 \times 10^4$ will satisfy the condition of minimum power level. Hence, there are four signal components that have a power of at least 10% of the power of the unmodulated signal. The components with frequencies $10^8 + 10^4$, $10^8 - 10^4$ have an amplitude equal to 57.67, whereas the signal components with frequencies $10^8 + 2 \times 10^4$, $10^8 - 2 \times 10^4$ have an amplitude equal to 35.28.

2) Using Carson's rule, the approximate bandwidth of the FM signal is

$$B_c = 2(\beta + 1) f_m = 2(2 + 1)10^4 = 6 \times 10^4 \text{ Hz}$$

Problem 4.10

1)

$$\begin{aligned} \beta_p &= k_p \max[|m(t)|] = 1.5 \times 2 = 3 \\ \beta_f &= \frac{k_f \max[|m(t)|]}{f_m} = \frac{3000 \times 2}{1000} = 6 \end{aligned}$$

2) Using Carson's rule we obtain

$$\begin{aligned} B_{\text{PM}} &= 2(\beta_p + 1) f_m = 8 \times 1000 = 8000 \\ B_{\text{FM}} &= 2(\beta_f + 1) f_m = 14 \times 1000 = 14000 \end{aligned}$$

3) The PM modulated signal can be written as

$$u(t) = \sum_{n=-\infty}^{\infty} A J_n(\beta_p) \cos(2\pi(10^6 + n10^3)t)$$

The next figure shows the amplitude of the spectrum for positive frequencies and for these components whose frequencies lie in the interval $[10^6 - 4 \times 10^3, 10^6 + 4 \times 10^3]$. Note that $J_0(3) = -.2601$, $J_1(3) = 0.3391$, $J_2(3) = 0.4861$, $J_3(3) = 0.3091$ and $J_4(3) = 0.1320$.

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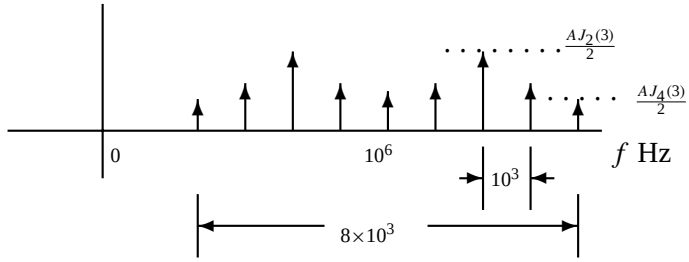
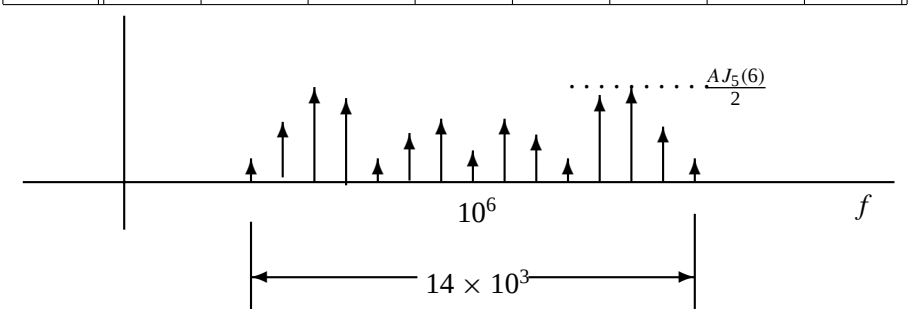


In the case of the FM modulated signal

$$\begin{aligned} u(t) &= A\cos(2\pi f_c t + \beta_f \sin(2000\pi t)) \\ &= \sum_{n=-\infty}^{\infty} AJ_n(6)\cos(2\pi(10^6 + n10^3)t + \phi_n) \end{aligned}$$

The next figure shows the amplitude of the spectrum for positive frequencies and for these components whose frequencies lie in the interval $[10^6 - 7\times10^3, 10^6 - 7\times10^3]$. The values of $J_n(6)$ for $n = 0, \ldots, 7$ are given in the following table.

n	0	1	2	3	4	5	6	7
$J_n(6)$	.1506	-.2767	-.2429	.1148	.3578	.3621	.2458	.1296



4) If the amplitude of $m(t)$ is decreased by a factor of two, then $m(t) = \cos(2\pi 10^3 t)$ and

$$\begin{aligned} \beta_p &= k_p \max[|m(t)|] = 1.5 \\ \beta_f &= \frac{k_f \max[|m(t)|]}{f_m} = \frac{3000}{1000} = 3 \end{aligned}$$

The bandwidth is determined using Carson's rule as

$$\begin{aligned} B_{\text{PM}} &= 2(\beta_p + 1) f_m = 5\times1000 = 5000 \\ B_{\text{FM}} &= 2(\beta_f + 1) f_m = 8\times1000 = 8000 \end{aligned}$$

The amplitude spectrum of the PM and FM modulated signals is plotted in the next figure for positive frequencies. Only those frequency components lying in the previous derived bandwidth are plotted. Note that $J_0(1.5) = .5118$, $J_1(1.5) = .5579$ and $J_2(1.5) = .2321$.

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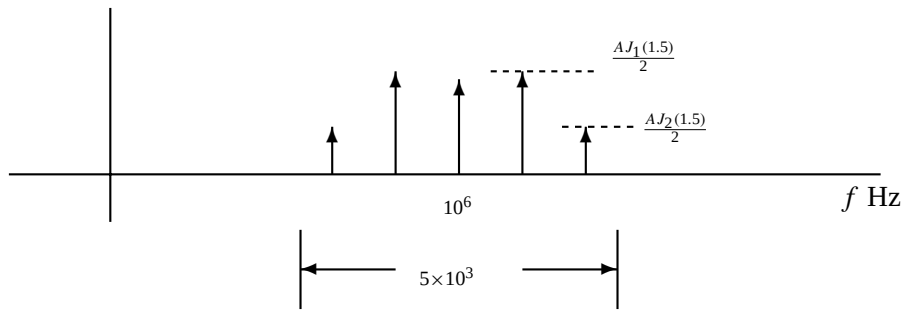

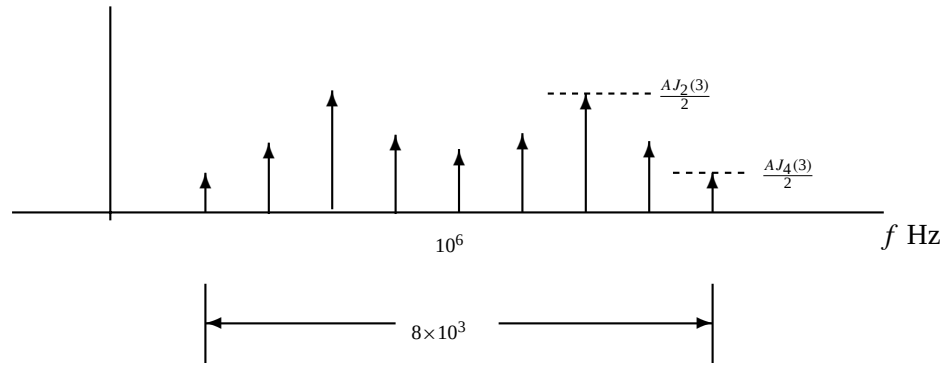

5) If the frequency of $m(t)$ is increased by a factor of two, then $m(t) = 2\cos(2\pi 2 \times 10^3 t)$ and

$$\begin{aligned}\beta_p &= k_p \max[|m(t)|] = 1.5 \times 2 = 3 \\ \beta_f &= \frac{k_f \max[|m(t)|]}{f_m} = \frac{3000 \times 2}{2000} = 3\end{aligned}$$

The bandwidth is determined using Carson's rule as

$$\begin{aligned}B_{\text{PM}} &= 2(\beta_p + 1) f_m = 8 \times 2000 = 16000 \\ B_{\text{FM}} &= 2(\beta_f + 1) f_m = 8 \times 2000 = 16000\end{aligned}$$

The amplitude spectrum of the PM and FM modulated signals is plotted in the next figure for positive frequencies. Only those frequency components lying in the previous derived bandwidth are plotted. Note that doubling the frequency has no effect on the number of harmonics in the bandwidth of the PM signal, whereas it decreases the number of harmonics in the bandwidth of the FM signal from 14 to 8.

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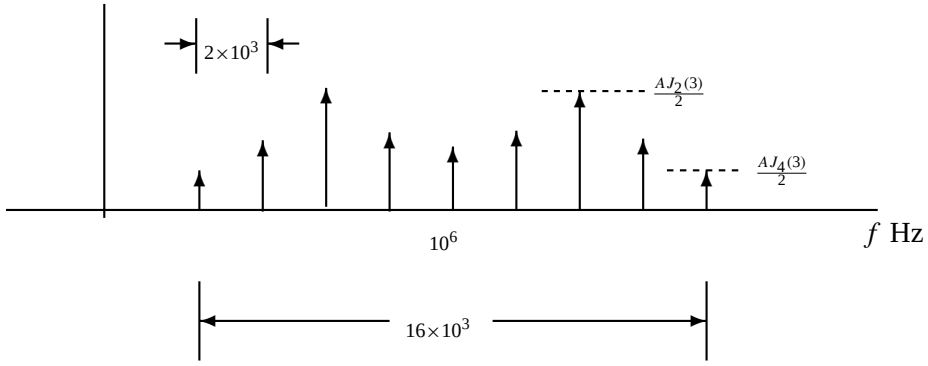

Problem 4.11

1) The PM modulated signal is

$$\begin{aligned} u(t) &= 100\cos(2\pi f_c t + \frac{\pi}{2}\cos(2\pi 1000t)) \\ &= \sum_{n=-\infty}^{\infty} 100 J_n\left(\frac{\pi}{2}\right)\cos(2\pi(10^8 + n10^3)t) \end{aligned}$$

The next table tabulates $J_n(\beta)$ for $\beta = \frac{\pi}{2}$ and $n = 0, \ldots, 4$.

n	0	1	2	3	4
$J_n(\beta)$	.4720	.5668	.2497	.0690	.0140

The total power of the modulated signal is $P_{\text{tot}} = \frac{100^2}{2} = 5000$. To find the effective bandwidth of the signal we calculate the index k such that

$$\sum_{n=-k}^{k} \frac{100^2}{2} J_n^2\left(\frac{\pi}{2}\right) \geq 0.99 \times 5000 \Longrightarrow \sum_{n=-k}^{k} J_n^2\left(\frac{\pi}{2}\right) \geq 0.99$$

By trial end error we find that the smallest index k is 2. Hence the effective bandwidth is

$$B_{\text{eff}} = 4 \times 10^3 = 4000$$

In the the next figure we sketch the magnitude spectrum for the positive frequencies.

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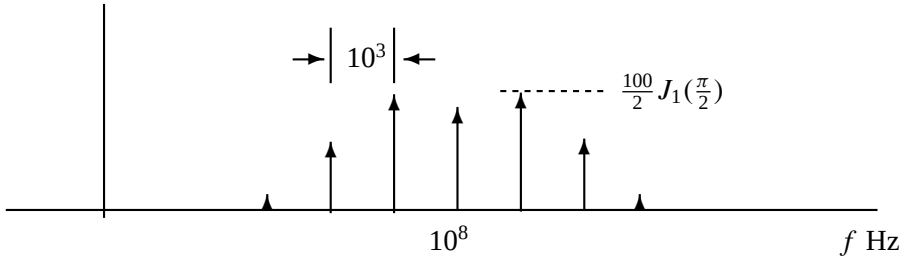

2) Using Carson's rule, the approximate bandwidth of the PM signal is

$$B_{\text{PM}} = 2(\beta_p + 1)f_m = 2(\frac{\pi}{2} + 1)1000 = 5141.6$$

As it is observed, Carson's rule overestimates the effective bandwidth allowing in this way some margin for the missing harmonics.

Problem 4.12

1) Assuming that $u(t)$ is an FM signal it can be written as

$$\begin{aligned} u(t) &= 100\cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{\infty} \alpha \cos(2\pi f_m \tau) d\tau) \\ &= 100\cos(2\pi f_c t + \frac{k_f \alpha}{f_m} \sin(2\pi f_m t)) \end{aligned}$$

Thus, the modulation index is $\beta_f = \frac{k_f \alpha}{f_m} = 4$ and the bandwidth of the transmitted signal

$$B_{\text{FM}} = 2(\beta_f + 1)f_m = 10 \text{ KHz}$$

2) If we double the frequency, then

$$u(t) = 100\cos(2\pi f_c t + 4\sin(2\pi 2 f_m t))$$

Using the same argument as before we find that $\beta_f = 4$ and

$$B_{\text{FM}} = 2(\beta_f + 1)2f_m = 20 \text{ KHz}$$

3) If the signal $u(t)$ is PM modulated, then

$$\beta_p = \Delta\phi_{\max} = \max[4\sin(2\pi f_m t)] = 4$$

The bandwidth of the modulated signal is

$$B_{\text{PM}} = 2(\beta_p + 1)f_m = 10 \text{ KHz}$$

4) If f_m is doubled, then $\beta_p = \Delta\phi_{\max}$ remains unchanged whereas

$$B_{\text{PM}} = 2(\beta_p + 1)2f_m = 20 \text{ KHz}$$

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Problem 4.13

1) If the signal $m(t) = m_1(t) + m_2(t)$ DSB modulates the carrier $A_c \cos(2\pi f_c t)$ the result is the signal

$$\begin{aligned} u(t) &= A_c m(t) \cos(2\pi f_c t) \\ &= A_c (m_1(t) + m_2(t)) \cos(2\pi f_c t) \\ &= A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \cos(2\pi f_c t) \\ &= u_1(t) + u_2(t) \end{aligned}$$

where $u_1(t)$ and $u_2(t)$ are the DSB modulated signals corresponding to the message signals $m_1(t)$ and $m_2(t)$. Hence, AM modulation satisfies the superposition principle.

2) If $m(t)$ frequency modulates a carrier $A_c \cos(2\pi f_c t)$ the result is

$$\begin{aligned} u(t) &= A_c \cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{\infty} (m_1(\tau) + m_2(\tau)) d\tau) \\ &\neq A_c \cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{\infty} m_1(\tau) d\tau) \\ &\quad + A_c \cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{\infty} m_2(\tau) d\tau) \\ &= u_1(t) + u_2(t) \end{aligned}$$

where the inequality follows from the nonlinearity of the cosine function. Hence, angle modulation is not a linear modulation method.

Problem 4.14

The transfer function of the FM discriminator is

$$H(s) = \frac{R}{R + Ls + \frac{1}{Cs}} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Thus,

$$|H(f)|^2 = \frac{4\pi^2 \left(\frac{R}{L}\right)^2 f^2}{\left(\frac{1}{LC} - 4\pi^2 f^2\right)^2 + 4\pi^2 (\frac{R}{L})^2 f^2}$$

As it is observed $|H(f)|^2 \leq 1$ with equality if

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Since this filter is to be used as a slope detector, we require that the frequency content of the signal, which is $[80 - 6, 80 + 6]$ MHz, to fall inside the region over which $|H(f)|$ is almost linear. Such a region can be considered the interval $[f_{10}, f_{90}]$, where f_{10} is the frequency such that $|H(f_{10})| = 10\% \max[|H(f)|]$ and f_{90} is the frequency such that $|H(f_{10})| = 90\% \max[|H(f)|]$.

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With $\max[|H(f)| = 1$, $f_{10} = 74 \times 10^6$ and $f_{90} = 86 \times 10^6$, we obtain the system of equations

$$4\pi^2 f_{10}^2 + \frac{50 \times 10^3}{L} 2\pi f_{10}[1 - 0.1^2]^{\frac{1}{2}} - \frac{1}{LC} = 0$$
$$4\pi^2 f_{90}^2 + \frac{50 \times 10^3}{L} 2\pi f_{90}[1 - 0.9^2]^{\frac{1}{2}} - \frac{1}{LC} = 0$$

Solving this system, we obtain

$$L = 14.98 \text{ mH} \quad C = 0.018013 \text{ pF}$$

Problem 4.15

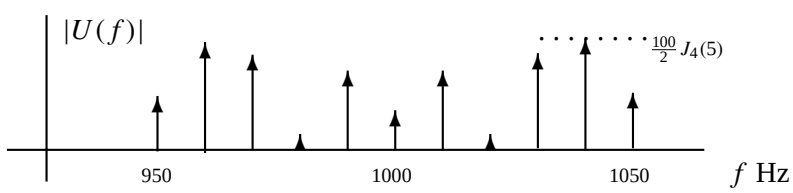
The case of $\phi(t) = \beta \cos(2\pi f_m t)$ has been treated in the text, the modulated signal is

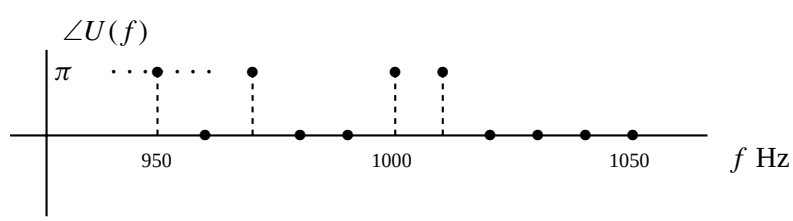
$$\begin{aligned} u(t) &= \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos(2\pi(f_c + n f_m)) \\ &= \sum_{n=-\infty}^{\infty} 100 J_n(5) \cos(2\pi(10^3 + n10)) \end{aligned}$$

The following table shows the values of $J_n(5)$ for $n = 0, \ldots, 5$.

n	0	1	2	3	4	5
$J_n(5)$	-.178	-.328	.047	.365	.391	.261

In the next figure we plot the magnitude and the phase spectrum for frequencies in the range [950, 1050] Hz. Note that $J_{-n}(\beta) = J_n(\beta)$ if n is even and $J_{-n}(\beta) = -J_n(\beta)$ if n is odd.





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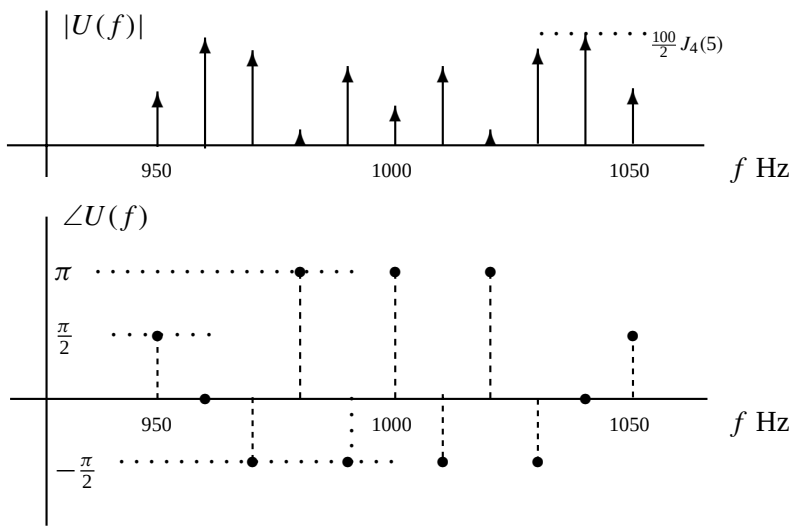
The Fourier Series expansion of $e^{j\beta \sin(2\pi f_m t)}$ is

$$\begin{aligned} c_n &= f_m \int_{\frac{1}{4f_m}}^{\frac{5}{4f_m}} e^{j\beta \sin(2\pi f_m t)} e^{-j2\pi n f_m t} dt \\ &= \frac{1}{2\pi} \int_0^{2\pi} e^{j\beta \cos u - jnu} e^{j\frac{n\pi}{2}} du \\ &= e^{j\frac{n\pi}{2}} J_n(\beta) \end{aligned}$$

Hence,

$$\begin{aligned} u(t) &= A_c \mathrm{Re}\left[\sum_{n=-\infty}^{\infty} c_n e^{j2\pi f_c t} e^{j2\pi n f_m t} \right] \\ &= A_c \mathrm{Re}\left[\sum_{n=-\infty}^{\infty} e^{j2\pi (f_c + n f_m)t + \frac{n\pi}{2}} \right] \end{aligned}$$

The magnitude and the phase spectra of $u(t)$ for $\beta = 5$ and frequencies in the interval [950, 1000] Hz are shown in the next figure. Note that the phase spectrum has been plotted modulo 2π in the interval $(-\pi, \pi]$.



Problem 4.16

The frequency deviation is given by

$$f_d(t) = f_i(t) - f_c = k_f m(t)$$

whereas the phase deviation is obtained from

$$\phi_d(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau$$

In the next figure we plot the frequency and the phase deviation when $m(t)$ is as in Fig. P-4.16 with $k_f = 25$.

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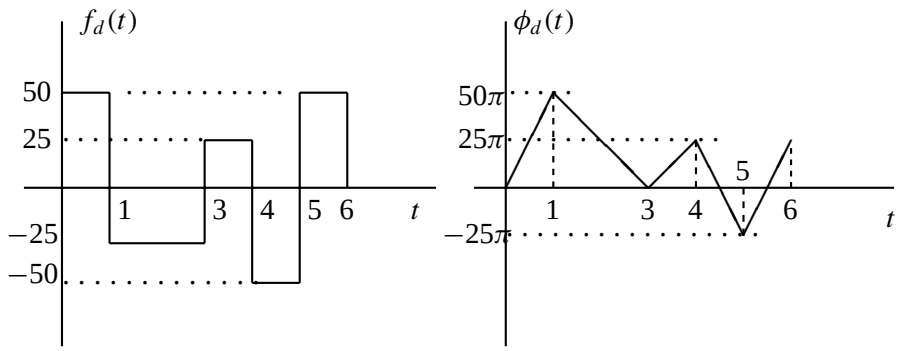

Problem 4.17

Using Carson's rule we obtain

$$B_c = 2(\beta+1)W = 2(\frac{k_f \max[|m(t)|]}{W}+1)W = \begin{cases} 20020 & k_f = 10 \\ 20200 & k_f = 100 \\ 22000 & k_f = 1000 \end{cases}$$

Problem 4.18

The modulation index is

$$\beta = \frac{k_f \max[|m(t)|]}{f_m} = \frac{10\times 10}{8} = 12.5$$

The output of the FM modulator can be written as

$$\begin{aligned} u(t) &= 10\cos(2\pi 2000t + 2\pi k_f \int_{-\infty}^{t} 10\cos(2\pi 8\tau)d\tau) \\ &= \sum_{n=-\infty}^{\infty} 10 J_n(12.5)\cos(2\pi(2000+n8)t+\phi_n) \end{aligned}$$

At the output of the BPF only the signal components with frequencies in the interval $[2000-32, 2000+32]$ will be present. These components are the terms of $u(t)$ for which $n=-4,\ldots,4$. The power of the output signal is then

$$\frac{10^2}{2}J_0^2(12.5) + 2\sum_{n=1}^{4}\frac{10^2}{2}J_n^2(12.5) = 50\times 0.2630 = 13.15$$

Since the total transmitted power is $P_{\text{tot}} = \frac{10^2}{2} = 50$, the power at the output of the bandpass filter is only 26.30% of the transmitted power.

Problem 4.19

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1) The instantaneous frequency is

$$f_i(t) = f_c + k_f m_1(t)$$

The maximum of $f_i(t)$ is

$$\max[f_i(t)] = \max[f_c + k_f m_1(t)] = 10^6 + 5 \times 10^5 = 1.5 \text{ MHz}$$

2) The phase of the PM modulated signal is $\phi(t) = k_p m_1(t)$ and the instantaneous frequency

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d}{dt}\phi(t) = f_c + \frac{k_p}{2\pi}\frac{d}{dt}m_1(t)$$

The maximum of $f_i(t)$ is achieved for t in [0, 1] where $\frac{d}{dt}m_1(t) = 1$. Hence, $\max[f_i(t)] = 10^6 + \frac{3}{2\pi}$.

3) The maximum value of $m_2(t) = \text{sinc}(2 \times 10^4 t)$ is 1 and it is achieved for $t = 0$. Hence,

$$\max[f_i(t)] = \max[f_c + k_f m_2(t)] = 10^6 + 10^3 = 1.001 \text{ MHz}$$

Since, $\mathcal{F}[\text{sinc}(2 \times 10^4 t)] = \frac{1}{2\times 10^4}\Pi(\frac{f}{2\times 10^4})$ the bandwidth of the message signal is $W = 10^4$. Thus, using Carson's rule, we obtain

$$B = 2(\frac{k_f \max[|m(t)|]}{W} + 1)W = 22 \text{ KHz}$$

Problem 4.20

Since 88 MHz $< f_c <$ 108 MHz and

$$|f_c - f_c'| = 2f_{\text{IF}} \quad \text{if } f_{\text{IF}} < f_{\text{LO}}$$

we conclude that in order for the image frequency f_c' to fall outside the interval [88, 108] MHZ, the minimum frequency f_{IF} is such that

$$2f_{\text{IF}} = 108 - 88 \Longrightarrow f_{\text{IF}} = 10 \text{ MHz}$$

If $f_{\text{IF}} = 10$ MHz, then the range of f_{LO} is $[88 + 10, 108 + 10] = [98, 118]$ MHz.

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